

Autoregressive Distributed Lag (ADL) Model

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The regressors may include lagged values of the dependent variable and current and lagged values of one or more explanatory variables. This model allows us to determine what the effects are of a change in a policy variable.

1. A simple model:

The ADL(1,1) model

$$y_t = m + \alpha_1 y_{t-1} + \beta_0 x_t + \beta_1 x_{t-1} + u_t,$$

where y_t and x_t are stationary variables, and u_t is a white noise.

The White-noise process: A sequence $\{u_t\}$ is a white-noise process if each value in the sequence has a mean of zero, a constant variance, and is serially uncorrelated.

The sequence $\{u_t\}$ is a white-noise process if for each period t ,

$$E(u_t) = E(u_{t-1}) = \dots = 0$$

$$E(u_t^2) = E(u_{t-1}^2) = \dots = \sigma^2$$

$$E(u_t u_{t-s}) = E(u_{t-j} u_{t-j-s}) = 0, \text{ for all } u.$$

2. Estimation:

If the values of x_t are treated as given, as being uncorrelated with u_t . OLS would be consistent. However, if x_t is simultaneously determined with y_t and $E(x_t u_t) \neq 0$, OLS would be inconsistent. As long as it can be assumed that the error term u_t is a white noise process, or more generally-is stationary and independent of x_t, x_{t-1}, \dots and y_t, y_{t-1}, \dots , the ADL models can be estimated consistently by ordinary least squares.

3. Interpretation of the dynamic effect:

We can invert the model as the lag polynomial in y as

$$y_t = (1 + \alpha_1 + \alpha_1^2 + \dots)m + (1 + \alpha_1 L + \alpha_1^2 L^2 + \dots)(\beta_0 x_t + \beta_1 x_{t-1} + u_t).$$

The current value of y depends on the current and all previous values of x and

u.

$$\frac{\partial y_t}{\partial x_t} = \beta_0,$$

This is referred as the impact multiplier.

The effect after one period

$$\frac{\partial y_{t+1}}{\partial x_t} = \beta_1 + \alpha_1 \beta_0,$$

The effect after two periods

$$\frac{\partial y_{t+2}}{\partial x_t} = \alpha_1 \beta_1 + \alpha_1^2 \beta_0,$$

The long-run multiplier (long-run effect) is $\frac{\beta_0 + \beta_1}{1 - \alpha_1}$ if $|\alpha_1| < 1$.

4. Reparameterization:

Substitute y_t and x_t with $y_{t-1} + \Delta y_t$ and $x_{t-1} + \Delta x_t$.

$$\Delta y_t = m + \beta_0 \Delta x_t - (1 - \alpha_1) y_{t-1} + (\beta_0 + \beta_1) x_{t-1} + u_t,$$

$$\Delta y_t = \beta_0 \Delta x_t - (1 - \alpha_1) \left[y_{t-1} - \frac{m}{1 - \alpha_1} - \frac{\beta_0 + \beta_1}{1 - \alpha_1} x_{t-1} \right] + u_t.$$

This is called the error correction model (ECM).

The current change in y is the sum of two components. The first is proportional to the current change in x . The second is a partial correction for the extent to which y_{t-1} deviated from the equilibrium value corresponding to x_{t-1} (the equilibrium error).

5. Generalizations:

The ADL(p, q) model:

$$A(L)y_t = m + B(L)x_t + u_t,$$

with

$$A(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p,$$

$$B(L) = \beta_0 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q.$$

The general ADL(p, q_1, q_2, \dots, q_k) model:

$$A(L)y_t = m + B_1(L)x_{1t} + B_2(L)x_{2t} + \dots + B_k(L)x_{kt} + u_t,$$

If $A(L) = 1$, the model does not contain any lags of y_t . It is called the distributed lag model.