Chapter Twenty-One

Cost Curves

Types of Cost Curves

- A total cost curve is the graph of a firm's total cost function.
- A variable cost curve is the graph of a firm's variable cost function.
- An average total cost curve is the graph of a firm's average total cost function.

Types of Cost Curves

- An average variable cost curve is the graph of a firm's average variable cost function.
- An average fixed cost curve is the graph of a firm's average fixed cost function.
- A marginal cost curve is the graph of a firm's marginal cost function.

Types of Cost Curves

- How are these cost curves related to each other?
- How are a firm's long-run and short-run cost curves related?

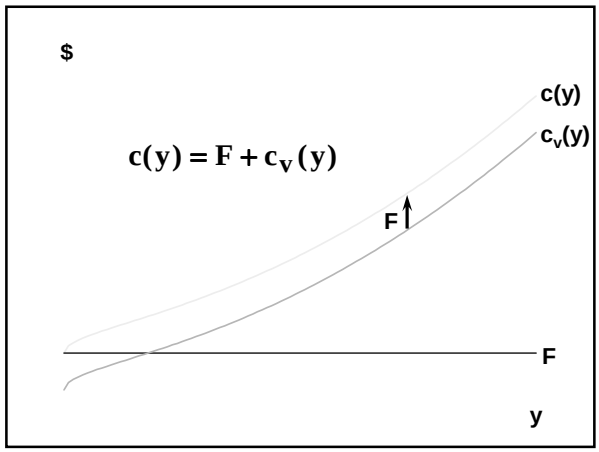
Fixed, Variable & Total Cost Functions

- F is the total cost to a firm of its short-run fixed inputs. F, the firm's fixed cost, does not vary with the firm's output level.
- $c_v(y)$ is the total cost to a firm of its variable inputs when producing y output units. $c_v(y)$ is the firm's variable cost function.
- $c_v(y)$ depends upon the levels of the fixed inputs.

Fixed, Variable & Total Cost Functions

- c(y) is the total cost of all inputs, fixed and variable, when producing y output units. c(y) is the firm's total cost function;

$$\mathbf{c(y) = F + c_v(y).}$$



Av. Fixed, Av. Variable & Av. Total Cost Curves

- The firm's total cost function is

$$\mathbf{c(y) = F + c_v(y).}$$

For y > 0, the firm's average total cost function is

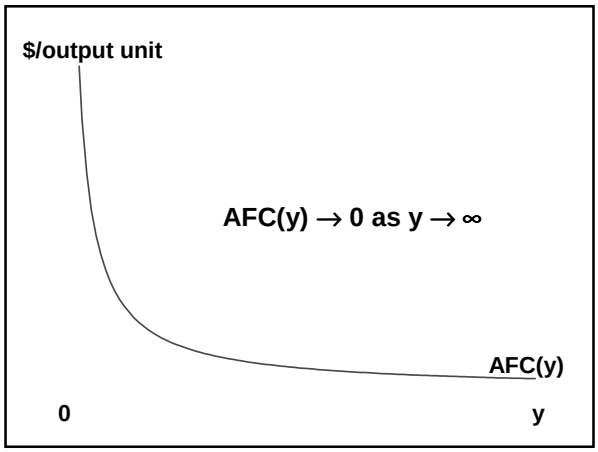
$$\mathbf{AC(y) = \frac{F}{y} + \frac{c_v(y)}{y}}$$
$$\mathbf{= AFC(y) + AVC(y).}$$

Av. Fixed, Av. Variable & Av. Total Cost Curves

- What does an average fixed cost curve look like?

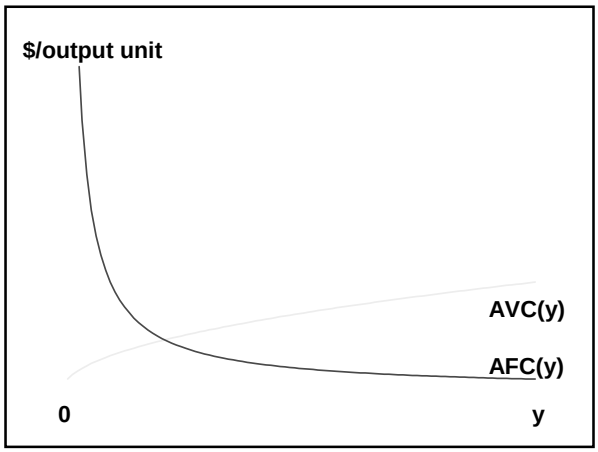
$$AFC(y) = \frac{F}{y}$$

- AFC(y) is a rectangular hyperbola so its graph looks like ...



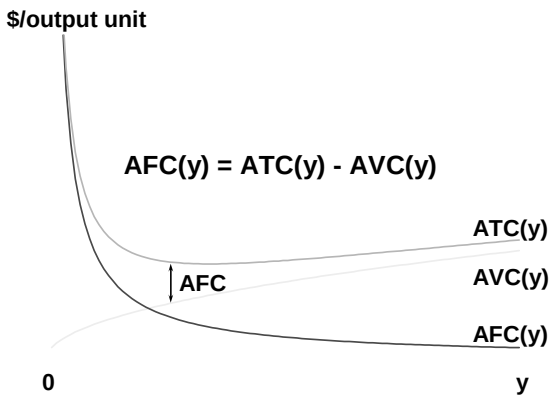
Av. Fixed, Av. Variable & Av. Total Cost Curves

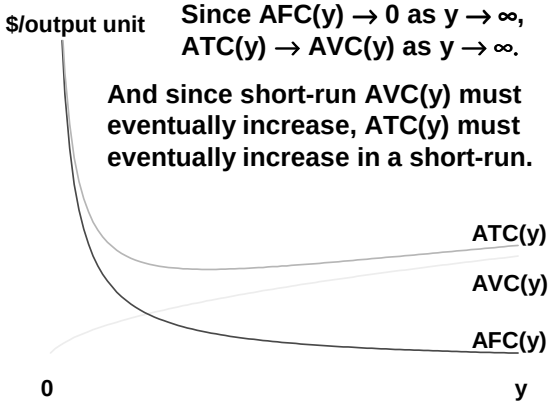
- In a short-run with a fixed amount of at least one input, the Law of Diminishing (Marginal) Returns must apply, causing the firm's average variable cost of production to increase eventually.



Av. Fixed, Av. Variable & Av. Total Cost Curves

- And ATC(y) = AFC(y) + AVC(y)





Marginal Cost Function

- Marginal cost is the rate-of-change of variable production cost as the output level changes. That is,

$$MC(y) = \frac{\partial c_V(y)}{\partial y}.$$

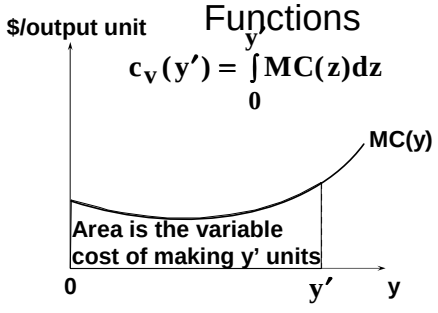
Marginal Cost Function

- The firm's total cost function is
$$c(y) = F + c_v(y)$$
and the fixed cost F does not change with the output level y, so
$$MC(y) = \frac{\partial c_v(y)}{\partial y} = \frac{\partial c(y)}{\partial y}.$$
- MC is the slope of both the variable cost and the total cost functions.

Marginal and Variable Cost Functions

- Since MC(y) is the derivative of $c_v(y)$, $c_v(y)$ must be the integral of MC(y). That is,
$$MC(y) = \frac{\partial c_v(y)}{\partial y}$$
$$\Rightarrow \quad c_v(y) = \int_0^y MC(z)dz.$$

Marginal and Variable Cost Functions



Marginal & Average Cost Functions

- How is marginal cost related to average variable cost?

Marginal & Average Cost Functions

Since $AVC(y) = \frac{c_V(y)}{y}$,

$$\frac{\partial AVC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c_V(y)}{y^2}.$$

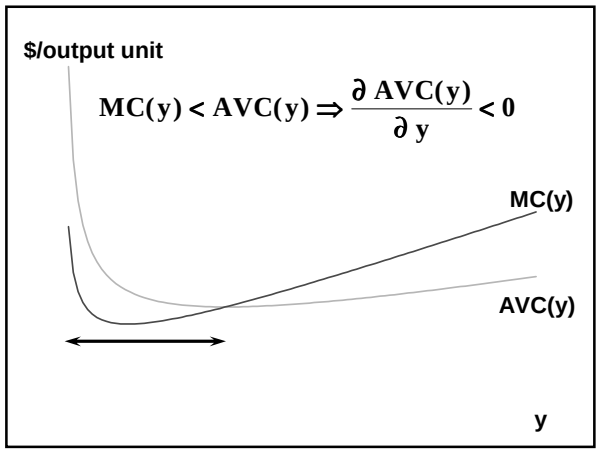
Therefore,

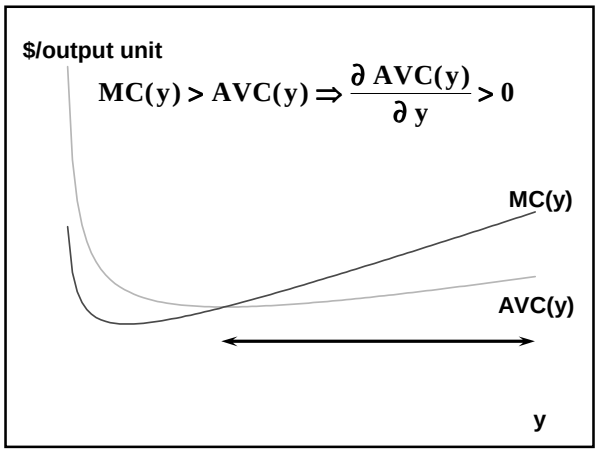
$$\frac{\partial AVC(y)}{\partial y} \gtreqless 0 \quad \text{as} \quad y \times MC(y) \gtreqless c_V(y).$$

$$\frac{\partial AVC(y)}{\partial y} \gtreqless 0 \quad \text{as} \quad MC(y) \gtreqless \frac{c_V(y)}{y} = AVC(y).$$

Marginal & Average Cost Functions

$$\frac{\partial AVC(y)}{\partial y} \gtreqless 0 \quad \text{as} \quad MC(y) \gtreqless AVC(y).$$

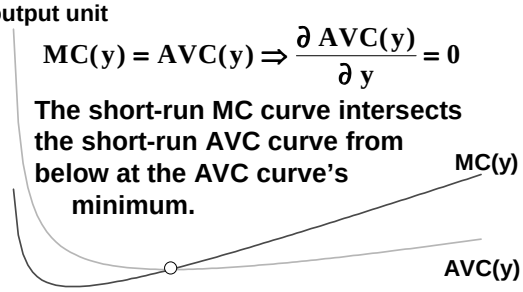




$/output unit

$$MC(y) = AVC(y) \Rightarrow \frac{\partial AVC(y)}{\partial y} = 0$$

The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.



Marginal & Average Cost Functions

Similarly, since $ATC(y) = \frac{c(y)}{y}$,

$$\frac{\partial ATC(y)}{\partial y} = \frac{y \times MC(y) - 1 \times c(y)}{y^2}.$$

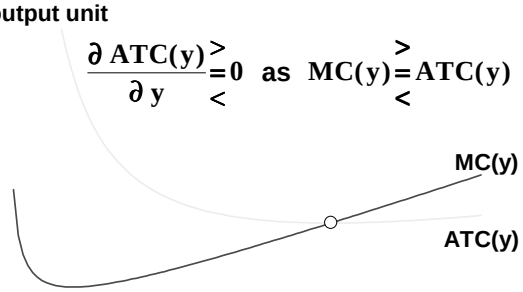
Therefore,

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad y \times MC(y) \begin{matrix} > \\ = \\ < \end{matrix} c(y).$$

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} \frac{c(y)}{y} = ATC(y).$$

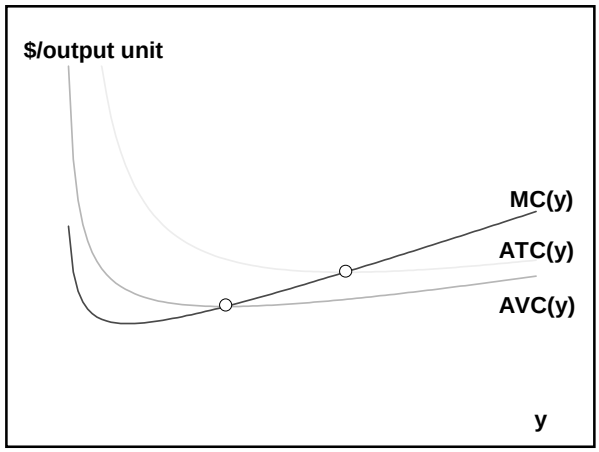
$/output unit

$$\frac{\partial ATC(y)}{\partial y} \begin{matrix} > \\ = \\ < \end{matrix} 0 \quad \text{as} \quad MC(y) \begin{matrix} > \\ = \\ < \end{matrix} ATC(y)$$



Marginal & Average Cost Functions

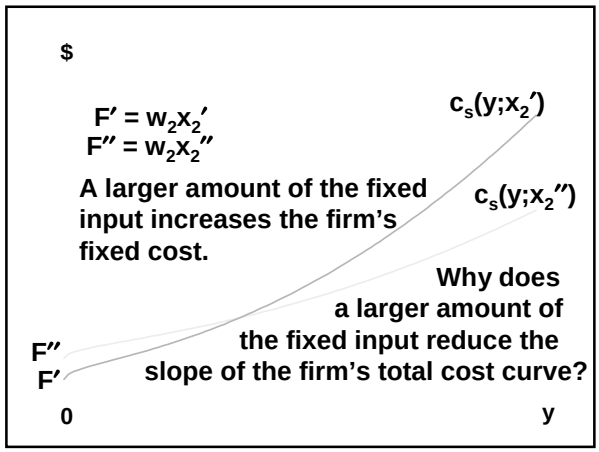
- The short-run MC curve intersects the short-run AVC curve from below at the AVC curve's minimum.
- And, similarly, the short-run MC curve intersects the short-run ATC curve from below at the ATC curve's minimum.



Short-Run & Long-Run Total Cost Curves

- A firm has a different short-run total cost curve for each possible short-run circumstance.
- Suppose the firm can be in one of just three short-runs;

$x_2 = x_2'$
or $x_2 = x_2''$ $\qquad x_2' < x_2'' < x_2'''$.
or $x_2 = x_2'''$.



Short-Run & Long-Run Total Cost Curves

MP_1 is the marginal physical productivity of the variable input 1, so one extra unit of input 1 gives MP_1 extra output units. Therefore, the extra amount of input 1 needed for 1 extra output unit is $1/MP_1$ units of input 1.

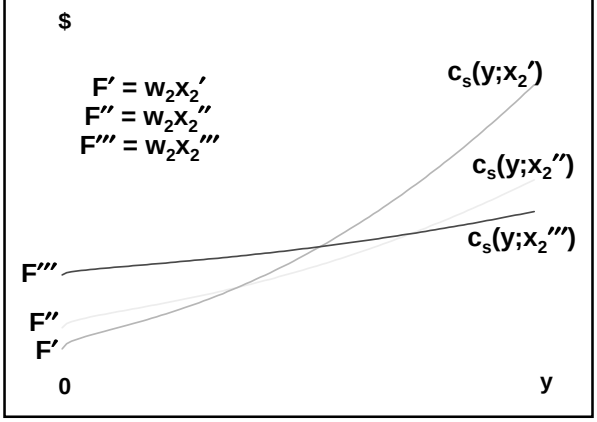
Each unit of input 1 costs w_1, so the firm's extra cost from producing one extra unit of output is $\mathbf{MC = \frac{w_1}{MP_1}}$.

Short-Run & Long-Run Total Cost Curves

$MC = \frac{w_1}{MP_1}$ **is the slope of the firm's total cost curve.**

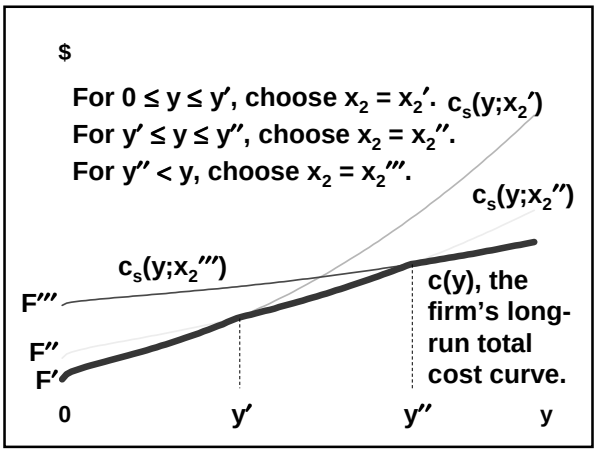
If input 2 is a complement to input 1 then MP_1 is higher for higher x_2.
Hence, MC is lower for higher x_2.

That is, a short-run total cost curve starts higher and has a lower slope if x_2 is larger.



Short-Run & Long-Run Total Cost Curves

- The firm has three short-run total cost curves.
- In the long-run the firm is free to choose amongst these three since it is free to select x_2 equal to any of x_2', x_2'', or x_2'''.
- How does the firm make this choice?

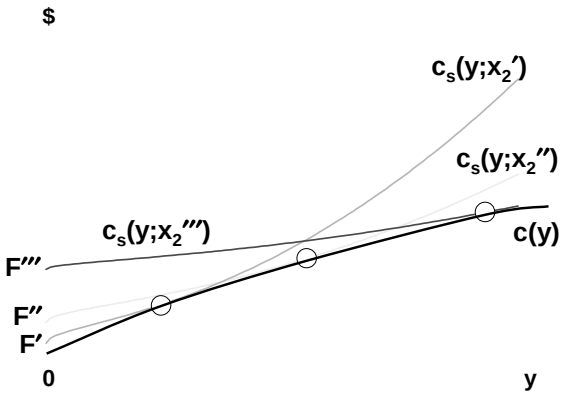


Short-Run & Long-Run Total Cost Curves

- The firm's long-run total cost curve consists of the lowest parts of the short-run total cost curves. The long-run total cost curve is the lower envelope of the short-run total cost curves.

Short-Run & Long-Run Total Cost Curves

- If input 2 is available in continuous amounts then there is an infinity of short-run total cost curves but the long-run total cost curve is still the lower envelope of all of the short-run total cost curves.

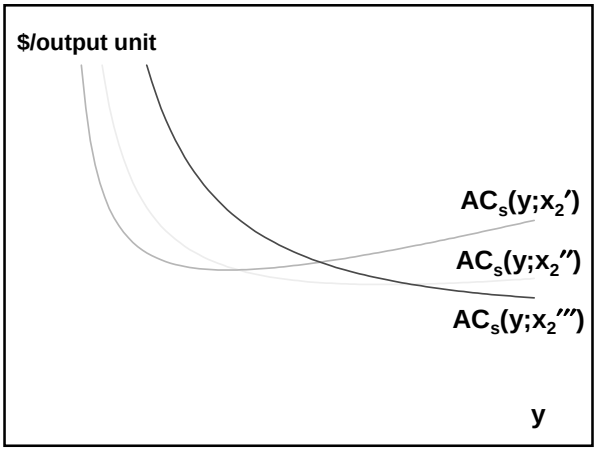


Short-Run & Long-Run Average Total Cost Curves

- For any output level y, the long-run total cost curve always gives the lowest possible total production cost.
- Therefore, the long-run av. total cost curve must always give the lowest possible av. total production cost.
- The long-run av. total cost curve must be the lower envelope of all of the firm's short-run av. total cost curves.

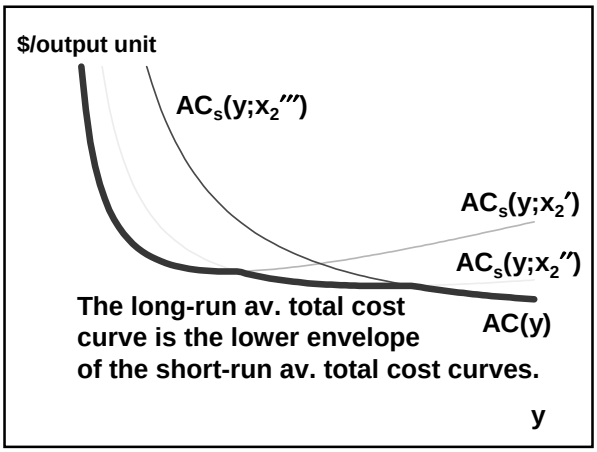
Short-Run & Long-Run Average Total Cost Curves

- E.g. suppose again that the firm can be in one of just three short-runs;
  $x_2 = x_2'$
  or $x_2 = x_2''$ $(x_2' < x_2'' < x_2''')$
  or $x_2 = x_2'''$
  then the firm's three short-run average total cost curves are ...



Short-Run & Long-Run Average Total Cost Curves

- The firm's long-run average total cost curve is the lower envelope of the short-run average total cost curves ...

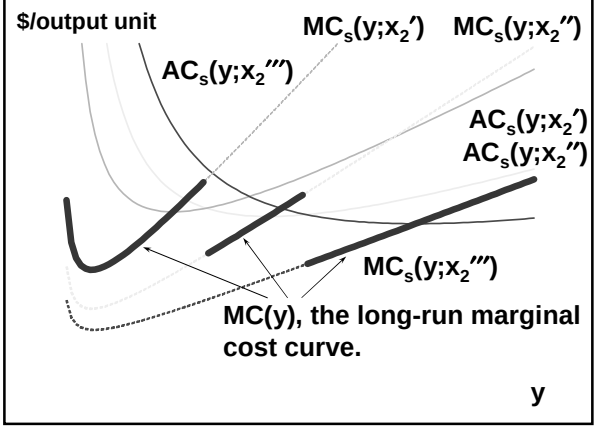


Short-Run & Long-Run Marginal Cost Curves

- Q: Is the long-run marginal cost curve the lower envelope of the firm's short-run marginal cost curves?
- A: No.

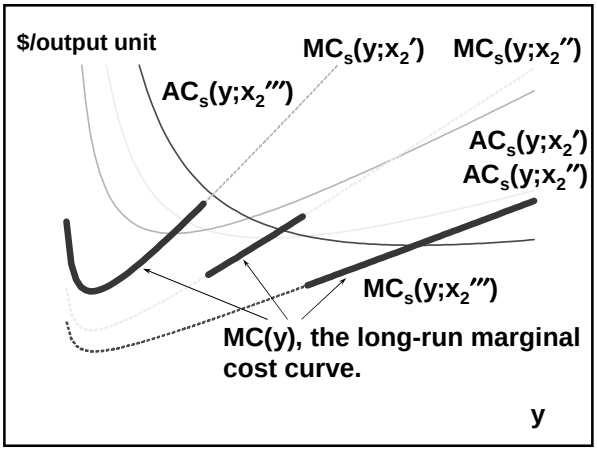
Short-Run & Long-Run Marginal Cost Curves

- The firm's three short-run average total cost curves are ...



Short-Run & Long-Run Marginal Cost Curves

- For any output level y > 0, the long-run marginal cost of production is the marginal cost of production for the short-run chosen by the firm.



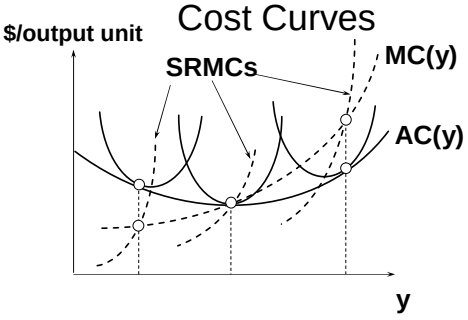
Short-Run & Long-Run Marginal Cost Curves

- For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- This is always true, no matter how many and which short-run circumstances exist for the firm.

Short-Run & Long-Run Marginal Cost Curves

- For any output level y > 0, the long-run marginal cost is the marginal cost for the short-run chosen by the firm.
- So for the continuous case, where x_2 can be fixed at any value of zero or more, the relationship between the long-run marginal cost and all of the short-run marginal costs is ...

Short-Run & Long-Run Marginal Cost Curves



◆For each y > 0, the long-run MC equals the MC for the short-run chosen by the firm.