Chapter Ten

Intertemporal Choice

Intertemporal Choice

- Persons often receive income in “lumps”; *e.g.* monthly salary.
- How is a lump of income spread over the following month (saving now for consumption later)?
- Or how is consumption financed by borrowing now against income to be received at the end of the month?

Present and Future Values

- Begin with some simple financial arithmetic.
- Take just two periods; 1 and 2.
- Let r denote the interest rate per period.

Future Value

- E.g., if r = 0.1 then \$100 saved at the start of period 1 becomes \$110 at the start of period 2.
- The value next period of \$1 saved now is the future value of that dollar.

Future Value

- Given an interest rate r the future value one period from now of \$1 is

$$FV = 1 + r.$$

- Given an interest rate r the future value one period from now of \$m is

$$FV = m(1 + r).$$

Present Value

- Suppose you can pay now to obtain \$1 at the start of next period.
- What is the most you should pay?
- \$1?
- No. If you kept your \$1 now and saved it then at the start of next period you would have \$(1+r) > \$1, so paying \$1 now for \$1 next period is a bad deal.

Present Value

- Q: How much money would have to be saved now, in the present, to obtain \$1 at the start of the next period?
- A: \$m saved now becomes \$m(1+r) at the start of next period, so we want the value of m for which

  $m(1+r) = 1$

  That is, $m = 1/(1+r)$,

  the present-value of \$1 obtained at the start of next period.

Present Value

- The present value of \$1 available at the start of the next period is

$$PV = \frac{1}{1 + r}.$$

- And the present value of \$m available at the start of the next period is

$$PV = \frac{m}{1 + r}.$$

Present Value

- E.g., if r = 0.1 then the most you should pay now for $1 available next period is

$$\mathbf{PV} = \frac{1}{1+0\cdot 1} = \$0\cdot 91.$$

- And if r = 0.2 then the most you should pay now for $1 available next period is

$$\mathbf{PV} = \frac{1}{1+0\cdot 2} = \$0\cdot 83.$$

The Intertemporal Choice Problem

- Let m_1 and m_2 be incomes received in periods 1 and 2.
- Let c_1 and c_2 be consumptions in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.

The Intertemporal Choice Problem

- The intertemporal choice problem: Given incomes m_1 and m_2, and given consumption prices p_1 and p_2, what is the most preferred intertemporal consumption bundle (c_1, c_2)?
- For an answer we need to know:
  - the intertemporal budget constraint
  - intertemporal consumption preferences.

The Intertemporal Budget Constraint

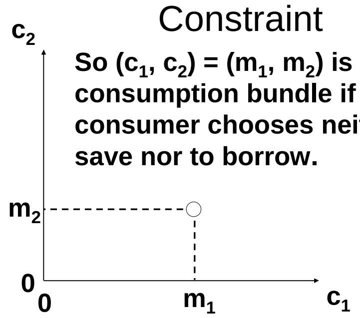
- To start, let's ignore price effects by supposing that

$$p_1 = p_2 = \$1.$$

The Intertemporal Budget Constraint

- Suppose that the consumer chooses not to save or to borrow.
- Q: What will be consumed in period 1?
- A: $c_1 = m_1$.
- Q: What will be consumed in period 2?
- A: $c_2 = m_2$.

The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

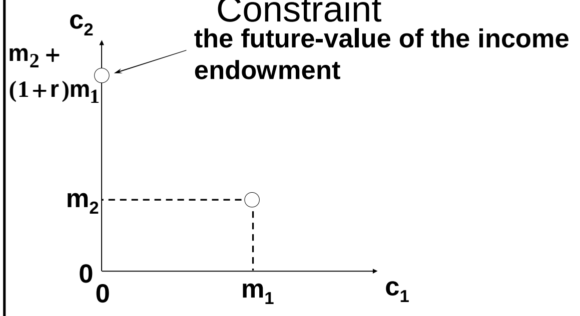
- Now suppose that the consumer spends nothing on consumption in period 1; that is, $c_1 = 0$ and the consumer saves $s_1 = m_1$.
- The interest rate is r.
- What now will be period 2's consumption level?

The Intertemporal Budget Constraint

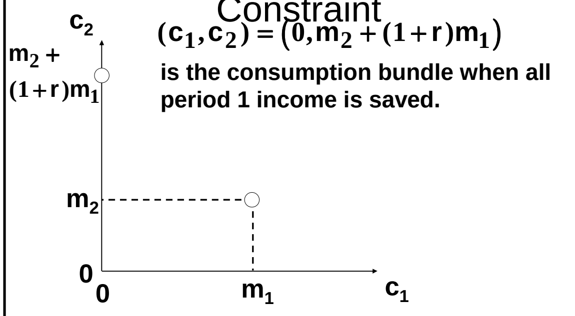
- Period 2 income is m_2.
- Savings plus interest from period 1 sum to $(1 + r)m_1$.
- So total income available in period 2 is $m_2 + (1 + r)m_1$.
- So period 2 consumption expenditure is

$$c_2 = m_2 + (1 + r)m_1$$

The Intertemporal Budget Constraint



The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

- Now suppose that the consumer spends everything possible on consumption in period 1, so $c_2 = 0$.
- What is the most that the consumer can borrow in period 1 against her period 2 income of \$$m_2$?
- Let b_1 denote the amount borrowed in period 1.

The Intertemporal Budget Constraint

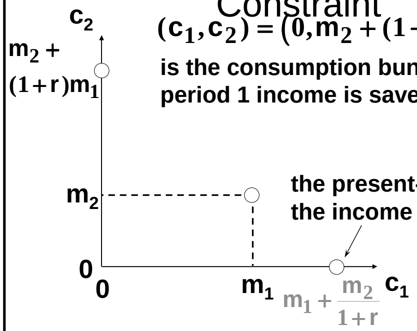
- Only \$$m_2$ will be available in period 2 to pay back \$$b_1$ borrowed in period 1.
- So $b_1(1 + r) = m_2$.
- That is, $b_1 = m_2 / (1 + r)$.
- So the largest possible period 1 consumption level is

$$c_1 = m_1 + \frac{m_2}{1+r}$$

The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when all period 1 income is saved.



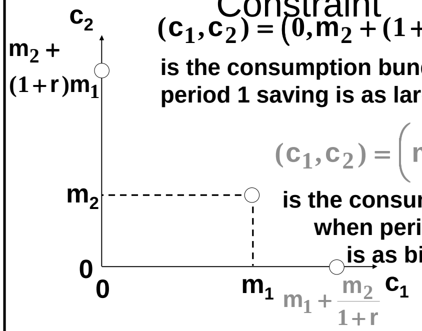
The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when period 1 saving is as large as possible.

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

is the consumption bundle when period 1 borrowing is as big as possible.



The Intertemporal Budget Constraint

- Suppose that c_1 units are consumed in period 1. This costs \$$c_1$ and leaves $m_1 - c_1$ saved. Period 2 consumption will then be

$$c_2 = m_2 + (1+r)(m_1 - c_1)$$

which is

$$c_2 = \underbrace{-(1+r)}_{\text{slope}}c_1 + \underbrace{m_2 + (1+r)m_1}_{\text{intercept}}.$$

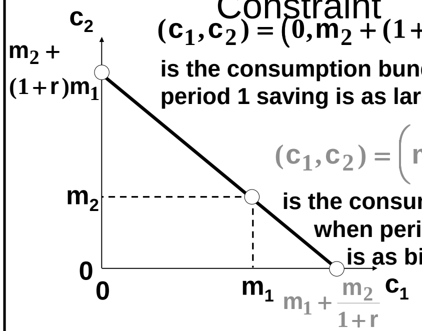
The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when period 1 saving is as large as possible.

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

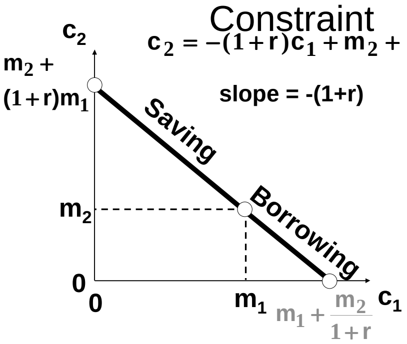
is the consumption bundle when period 1 borrowing is as big as possible.



The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$

slope = -(1+r)



The Intertemporal Budget Constraint

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

is the "future-valued" form of the budget constraint since all terms are in period 2 values. This is equivalent to

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

which is the "present-valued" form of the constraint since all terms are in period 1 values.

The Intertemporal Budget Constraint

- Now let's add prices p_1 and p_2 for consumption in periods 1 and 2.
- How does this affect the budget constraint?

Intertemporal Choice

- Given her endowment (m_1,m_2) and prices p_1, p_2 what intertemporal consumption bundle (c_1^*,c_2^*) will be chosen by the consumer?
- Maximum possible expenditure in period 2 is $$m_2 + (1+r)m_1$$ so maximum possible consumption in period 2 is $$c_2 = \frac{m_2 + (1+r)m_1}{p_2}.$$

Intertemporal Choice

- Similarly, maximum possible expenditure in period 1 is

$$m_1 + \frac{m_2}{1+r}$$

so maximum possible consumption in period 1 is

$$c_1 = \frac{m_1 + m_2/(1+r)}{p_1}.$$

Intertemporal Choice

- Finally, if c_1 units are consumed in period 1 then the consumer spends p_1c_1 in period 1, leaving $m_1 - p_1c_1$ saved for period 1. Available income in period 2 will then be

$$m_2 + (1+r)(m_1 - p_1c_1)$$

so

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1).$$

Intertemporal Choice

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1)$$

rearranged is

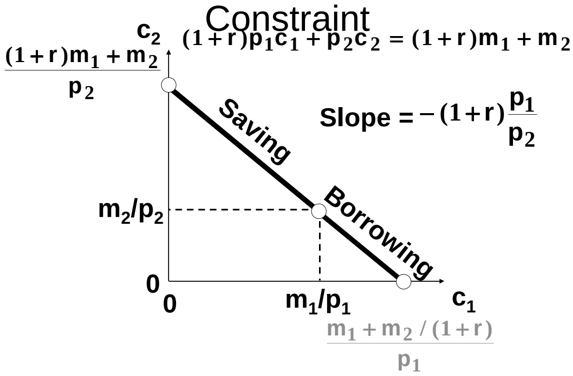
$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2.$$

This is the "future-valued" form of the budget constraint since all terms are expressed in period 2 values. Equivalent to it is the "present-valued" form

$$p_1c_1 + \frac{p_2}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

where all terms are expressed in period 1 values.

The Intertemporal Budget Constraint



Price Inflation

- Define the inflation rate by π where

$$p_1(1+\pi) = p_2.$$

- For example,
  π = 0.2 means 20% inflation, and
  π = 1.0 means 100% inflation.

Price Inflation

- We lose nothing by setting $p_1=1$ so that $p_2 = 1+\pi$.
- Then we can rewrite the budget constraint

as
$$p_1c_1 + \frac{p_2}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$
$$c_1 + \frac{1+\pi}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

Price Inflation

$$c_1 + \frac{1+\pi}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

rearranges to

$$c_2 = -\frac{1+r}{1+\pi}c_1 + (1+\pi)\left(\frac{m_1}{1+r} + m_2\right)$$

so the slope of the intertemporal budget constraint is $-\frac{1+r}{1+\pi}$.

Price Inflation

- When there was no price inflation ($p_1=p_2=1$) the slope of the budget constraint was -(1+r).
- Now, with price inflation, the slope of the budget constraint is $-(1+r)/(1+\pi)$. This can be written as

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

ρ is known as the real interest rate.

Real Interest Rate

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

gives

$$\rho = \frac{r-\pi}{1+\pi}.$$

For low inflation rates ($\pi \approx 0$), $\rho \approx r - \pi$. For higher inflation rates this approximation becomes poor.

Real Interest Rate

r	0.30	0.30	0.30	0.30	0.30
π	0.0	0.05	0.10	0.20	1.00
r - π	0.30	0.25	0.20	0.10	-0.70
ρ	0.30	0.24	0.18	0.08	-0.35

Comparative Statics

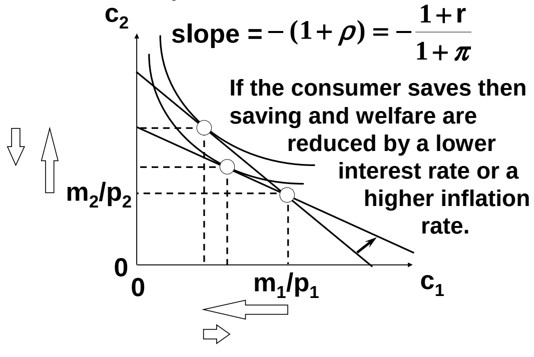
- The slope of the budget constraint is

$$-(1+\rho) = -\frac{1+r}{1+\pi}.$$

- The constraint becomes flatter if the interest rate r falls or the inflation rate π rises (both decrease the real rate of interest).

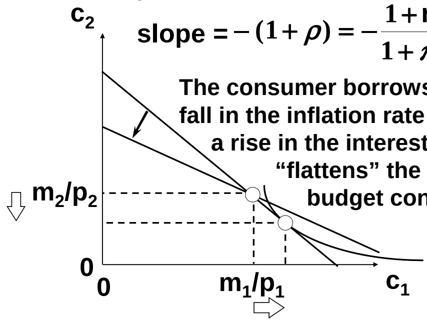
Comparative Statics

slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



Comparative Statics

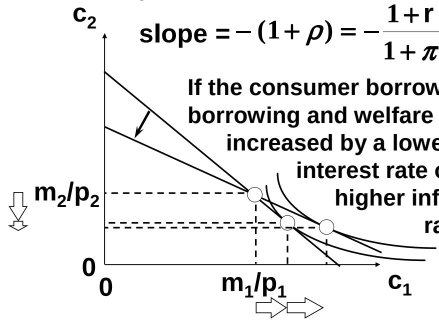
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



The consumer borrows. A fall in the inflation rate or a rise in the interest rate "flattens" the budget constraint.

Comparative Statics

slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



If the consumer borrows then borrowing and welfare are increased by a lower interest rate or a higher inflation rate.

Valuing Securities

- A financial security is a financial instrument that promises to deliver an income stream.
- E.g.; a security that pays
  - \$$m_1$ at the end of year 1,
  - \$$m_2$ at the end of year 2, and
  - \$$m_3$ at the end of year 3.
- What is the most that should be paid now for this security?

Valuing Securities

- The security is equivalent to the sum of three securities;
  - the first pays only \$$m_1$ at the end of year 1,
  - the second pays only \$$m_2$ at the end of year 2, and
  - the third pays only \$$m_3$ at the end of year 3.

Valuing Securities

- The PV of $\$m_1$ paid 1 year from now is $m_1 / (1+r)$
- The PV of $\$m_2$ paid 2 years from now is $m_2 / (1+r)^2$
- The PV of $\$m_3$ paid 3 years from now is $m_3 / (1+r)^3$
- The PV of the security is therefore

$$m_1 / (1+r) + m_2 / (1+r)^2 + m_3 / (1+r)^3.$$

Valuing Bonds

- A bond is a special type of security that pays a fixed amount \$x for T years (its maturity date) and then pays its face value \$F.
- What is the most that should now be paid for such a bond?

Valuing Bonds

$$\frac{\$x}{1+r} \quad \frac{\$x}{(1+r)^2} \quad \frac{\$x}{(1+r)^3} \qquad \frac{\$x}{(1+r)^{T-1}} \quad \frac{\$F}{(1+r)^T}$$

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \cdots + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}.$$

Valuing Bonds

- Suppose you win a State lottery. The prize is \$1,000,000 but it is paid over 10 years in equal installments of \$100,000 each. What is the prize actually worth?

Valuing Bonds

$$\text{PV} = \frac{\$100{,}000}{1+0\cdot 1} + \frac{\$100{,}000}{(1+0\cdot 1)^2} + \text{K} + \frac{\$100{,}000}{(1+0\cdot 1)^{10}}$$

$$= \$614{,}457$$

is the actual (present) value of the prize.

Valuing Consols

- A consol is a bond which never terminates, paying \$x per period forever.
- What is a consol's present-value?

Valuing Consols

End of Year	1	2	3	…	t	…
Income Paid	\$x	\$x	\$x	\$x	\$x	\$x
Present -Value	$\frac{\$x}{1+r}$	$\frac{\$x}{(1+r)^2}$	$\frac{\$x}{(1+r)^3}$	…	$\frac{\$x}{(1+r)^t}$	…

$$\text{PV} = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \text{K} + \frac{x}{(1+r)^t} + \text{K}\,.$$

Valuing Consols

$$\text{PV} = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \text{K}$$

$$= \frac{1}{1+r}\left[x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \text{K}\right]$$

$$= \frac{1}{1+r}[x + \text{PV}].$$

Solving for PV gives

$$\text{PV} = \frac{x}{r}.$$

Valuing Consols

E.g. if r = 0.1 now and forever then the most that should be paid now for a console that provides $1000 per year is

$$PV = \frac{x}{r} = \frac{\$1000}{0 \cdot 1} = \$10{,}000.$$