淡江大學 102 學年度第 2 學期財金二 B 財務數量方法第 1 次小考

系級：__________ 學號：__________ 姓名：__________

<沒寫計算過程不予計分，作弊以零分計算>

1. A firm has the following total-cost and demand functions:

$$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$$

$$Q = 100 - P$$

(a) Write out the total-revenue function R in terms of Q.

$Q = 100 - P \Rightarrow P = 100 - Q$

$R = P \times Q = (100 - Q) \times Q = \underline{100Q - Q^2}$ #

(b) Formulate the total-profit function π in terms of Q.

$\pi = R - C = 100Q - Q^2 - \frac{1}{3}Q^3 + 7Q^2 - 111Q - 50$

$= \underline{-\frac{1}{3}Q^3 + 6Q^2 - 11Q - 50}$ #

(c) Find the profit-maximizing level of output Q^*

$\frac{d\pi}{dQ} = -Q^2 + 12Q - 11 = 0$

$= -(Q^2 - 12Q + 11) = 0$

$= -(Q - 11)(Q - 1) = 0$

$Q^* = 1$ or 11

$\frac{d^2\pi}{dQ^2}\Big|_{Q^*=1} = -2Q + 12 = 10 > 0$ (不合)

$\frac{d^2\pi}{dQ^2}\Big|_{Q^*=11} = -2Q + 12 = -10 < 0$

(d) What is the maximum profit?

$\pi = -\frac{1}{3}(11)^3 + 6 \times (11)^2 - 11 \times (11) - 50 = \underline{111.33}$ #

2. Find the **first six terms** of the Maclaurin series (i.e., n=5, and let $x_0 = 0$) for:

(a) $f(x) = \frac{1}{1+x} = (1+x)^{-1}$

$f(0) = \frac{1}{1+0} = 1$

$f'(0) = (-1)(1+x)^{-2} = -1$

$f''(0) = 2(1+x)^{-3} = 2$

$f'''(0) = -6(1+x)^{-4} = -6$

$f^4(0) = 24(1+x)^{-5} = 24$

$f^5(0) = -120(1+x)^{-6} = -120$

$f(x) = \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4 + \frac{f^5(0)}{5!}x^5$

$= \underline{1 - x + x^2 - x^3 + x^4 - x^5}$ #

(b) $f(x) = \frac{1+x}{1-x}$

$f(0) = \frac{1+x}{1-x} = 1$

$f'(0) = \frac{(1-x)-(1+x)(-1)}{(1-x)^2} = \frac{1-x+1+x}{(1-x)^2} = \frac{2}{(1-x)^2} = 2(1-x)^{-2} = 2$

$f''(0) = -4(1-x)^{-3}(-1) = 4$

$f'''(0) = -12(1-x)^{-4}(-1) = 12$

$f^4(0) = -48(1-x)^{-5}(-1) = 48$

$f^5(0) = -240(1-x)^{-6}(-1) = 240$

$f(x) = \frac{f(0)}{0!} + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4 + \frac{f^5(0)}{5!}x^5$

$= 1 + 2x + 2x^2 + 2x^3 + 2x^4 + 2x^5$ #

3. Find the stationary values of the following functions:

(Use the Nth-derivative test to determine the exact nature of theses stationary values.)

(a) $y = (x-1)^2 + 16$

$y' = 2(x-1) = 0 \quad x^* = 1$

$y''(1) = 2 > 0$ ∵ n為偶數且 $y''(1) > 0$ ∴ $x=1, y=16$ 為最小值 #

(b) $y = (5-2x)^3 + 8$

$y' = 3(5-2x)^2(-2) = (-6)(5-2x)^2 = 0, \ x^* = 5/2$

$y''(5/2) = (-12)(5-2x)(-2) = 24(5-2x) = 120-48x = 0$

$y'''(5/2) = -48 < 0$ ∵ n為奇數 ∴ $x = 5/2, y = 8$ 為反曲點 #

(c) $y = (x-2)^6$

$y' = 6(x-2)^5 = 0, \ x^* = 2$

$y'' = 30(x-2)^4 = 0$

$y''' = 120(x-2)^3 = 0$

$y^4 = 360(x-2)^2 = 0$

$y^5 = 720(x-2) = 0$

$y^6 = 720 > 0$

∵ n為偶數, 且 $y^6(2) = 720 > 0$

∴ $x=2, y=0$ 為最小值 #

4. Evaluate the following:

$$(e^{\ln 5})! - \ln e^{3!} + (\log_{10} e)(\log_e 10{,}000)$$

$= 5! - 3! + \log_{10} 10{,}000$

$= 120 - 6 + 4$

$= \underline{118}$ #

5. Find the inverse function of $y = ab^{ct}$

$\ln y = \ln a + ct \ln b$

$\ln y - \ln a = ct \ln b$

$\Rightarrow t = \dfrac{\ln y - \ln a}{c \ln b}$ #

6. Write and exponential expression for the value:

(a) \$70, compounded continuously at the interest rate of 4% for 5 years.

$V = 70e^{0.04\times 5} = 70e^{0.2} = \underline{85.50}$ #

(b) \$580, compounded continuously at the interest rate of 8% for 4 years.

$V = 580e^{0.08\times 4} = 580e^{0.32} = \underline{798.73}$ #

7. What is the instantaneous rate of growth of y in each of the following?

(a) $y = e^{0.07t}$

$\dfrac{dy}{dt} = 0.07e^{0.07t}$

$\dfrac{dy/dt}{y} = \dfrac{0.07e^{0.07t}}{e^{0.07t}}$

$= \underline{0.07}$ #

(b) $y = 15e^{0.08t}$

$\dfrac{dy}{dt} = (15)(0.08)e^{0.08t}$

$\dfrac{dy/dt}{y} = \dfrac{(15)(0.08)e^{0.08t}}{15e^{0.08t}}$

$= \underline{0.08}$ #